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Electric and Scalar Charged Fluid Spheres in General Relativity

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Abstract

This research paper explores the theoretical frame work of general relativity to investigate the properties of electrically and scalar charged fluid spheres. By incorporating the principles of general relativity, electromagnetism, and scalar fields, this study aims to understand the behavior of charged fluid spheres within the context of Einstein's field equations. In this paper we have investigated the interior solution of an electric and scalar charged fluid sphere in general relativity under certain assumptions.

If we set α=β= γ =1, then we get the solution.

1. Introduction:

An interior solution of electric and scalar charged static 'dust' sphere has been studied by Teixeira et. al.[25] and it has been shown that the geometric mass of an electric and scalar charged sphere is m=$\sqrt{(q^2-\tau b^2)}$ where q is the electric charge, b is the scalar charge and $\tau=\pm 1$ Som et. al. [24] have found Reissner Nordstrom solution from the Schwarzchild solution by a coordinate transformation and have shown that the geometric mass of a charged sphere m=$\sqrt{({m_s}^2+q^2)}$ where m_s is the Schwarzschild mass and q is the charge of the sphere. Florides [9] has shown that the geometric mass of a charged sphere is m=$\psi(a)+\omega(a)$, where $\psi(a)$ is the contribution from the mass density and $\omega(a)$ is that form electric charge. Paul [18] has obtained an interior solution of electric and scalar charged fluid sphere and has shown that the geometric mass of the sphere has contribution from its mass density and scalar and electric charges.

In this paper we have investigated the interior solution of an electric and charged fluid sphere in general relativity under certain assumptions. If we set λ= ζ = τ =1, then we get the solution due to Yadav et. al. [23].

2. Solution of the Field Equations

We take the symmetrically symmetric line element in the form

$ds^2=e^{2M}dt^2-e^{2L}dr^2-r^2e^{L-M}d\theta^2-$
$r^2e^{L-M}\sin^2\theta\, d\o^2$ ……………………………..(2.1)

Where L=L(r) and M=M(r). Here r, θ, ø and t are numbered 1, 2, 3 and 4 respectively.

Einstein-Maxwell scalar field equations are

$R_\psi^v=-8\pi\left(T_\psi^v-\frac{1}{2}\delta_\psi^v T\right)$,…………………………(2.2)

$T_\psi^v = (\rho+p)u_\psi u^v-\delta_\psi^v p + \frac{1}{4\pi}\left[-F^{VL}F_{vL}+\right.$
$14\delta\psi$
$vFL\eta FL\eta+K\psi V$,…………………………….(2.3)

$4\pi V K_\psi^V = S^{;V}S_{;V}-\frac{1}{2}\delta_\psi^v S^{;L}S_{;L}$, ………………(2.4)

$F_{;V}^{\psi V}=4\pi\sigma u^\psi$, ………………………………..(2.5)

$F_{\psi v;L}=0$, …………………………...(2.6)

$S_{;v}^{;\psi}=-4\pi V_\eta$ …………………………………….(2.7)

Where ρ, p, σ are the mass density, pressure charged density and scalar charged density respectively. S is the scalar field. The matter is at rest in the coordinate system of (2.1) so that $u^\psi(g_{44})$.

The electric field is such that only $F_{41}=+H_1$ exists along radial direction where H is the electric potential. The equation (2.2) with the help of (2.1) gives

$M_{11}+\frac{2M_1}{r}=4\pi((\rho+3p))e^{2L}+e^{-2M}H_1^2$, …(2.8)

$L_{11}-\frac{L_1^2}{2}-L_1M_1+\frac{3}{2}M_1^2-\frac{2M_1}{r}=-4\pi(\rho-p)e^{2L}+$
$e^{-2M}H_1^2-2VS_1^2$, ……………………………(2.9)

$\frac{1}{2}M_{11}-\frac{1}{2}L_{11}+\frac{M_1}{r}-\frac{L_1}{L}-\frac{1}{r^2}+\frac{1}{r^2}e^{M+L}=4\pi(\rho-p)e^{2L}$
$+e^{-2M}H_1^2$ ………………………………….(2.10)

$\frac{d}{dr}(r^2e^{-2M}H_1)=-4\pi\sigma r^2e^{2L-M}$,………….(2.11)

$\frac{d}{dr}(r^2S_1)=4\pi V\eta r^2e^{2L}$ ………………….….(2.12)

Here we have five equations and eight variables so the system is indeterminate.

To make the system determinate we require three more equations.
For this we assume

$2L = Ar^{\lambda+1}$ ……………………………….……(2.13)
$2M = Br^{\zeta+1}$
$S = Cr^{\tau+1}$

Where A , B , C , λ, ζ , and τ are constants. The first two assumptions of (2.13) ensure flatness at the center and the third makes the scalar field zero at r = 0.

Now with the help of (2.13) equations (2.8) to (2.12) give

$16\pi p = \left[\frac{(\zeta+1)(\zeta+2)r^{\zeta-1}B}{4} + \frac{(\lambda+1)(\lambda+2)r^{\lambda-1}A}{4} -\right.$
1r2eArλ+1+Brζ+1 –1 × e−Arλ+1 ………(2.14)

$8\pi\rho = \left[\frac{(\zeta+1)\, r^{\zeta-1}\,(3\zeta+14)B}{8} - \frac{(\lambda+1)(5\lambda+2)A}{8} -\right.$
18r23ζ2+6ζ+3B2r2ζ+1−λ2+2λ+1A2r2λ+1−2λ+1ζ+1ABrλ+ζ+2+16V(τ+1)2C2r2τ+1+12r2eArλ+1+Brζ+12−1
$\times e^{-Ar^{\lambda+1}}$
………………………………………………..(2.15)

$H_1^2 e^{-2M} = \frac{(\zeta+1)(\zeta-2)r^{(\zeta-1)}}{8} B + \frac{(\lambda+1)(\lambda-2)r^{(\lambda-1)}}{8}$
$+ \frac{1}{16r^2}\{(3\zeta^2 + 6\zeta + 3)\, r^2B^2 - (\lambda+1)^2 r^{2(\lambda+1)}A^2 -$
$2(\lambda+1)(\zeta+1)r^{\lambda+\zeta+2}AB + 16V(\tau+1)^2C^2r^{2(\tau+1)}\}$
$+ \frac{1}{2r^2}(e^{\frac{Ar^{\lambda+1}Br^{\zeta+1}}{2}} - 1)$,
………………………(2.16)

$4\pi\sigma = \left[\frac{dC^{41}}{dr} + \frac{2}{r}C^{41}(\lambda+1)Ar^{\lambda}C^{41}\right] e^{\frac{Br^{\lambda+1}}{2}}$ …(2.17)

$4\pi\eta = (\tau+1)(\tau+2)r^{(\tau-1)}V^{-1}Ce^{Ar^{(\tau+1)}}$ …(2.18)

Now at r=0, we see that ρ, p, σ, η i.e. mass density, pressure, charge density and scalar charge density will vanish for λ , ζ ,τ ≥ 3.

For λ = ζ = τ=0, at the center r=0,

$2L = Ar$ ………………..(2.19)
$2M = Br$
$S = Cr$

Where A ,B ,C are constants. The first two assumptions of equation (2.19) ensure flatness at the center and the third makes the scalar field zero at r=0.

Now with the help of equation (2.12) equations (2.8) to (2.12) give

$16\pi p = \frac{e^{-Ar}}{r^2}\left[(A+B)\frac{r}{2} - e^{(A+B)r} + 1\right]$ ……(2.20)

$8\pi\rho = \frac{e^{-Ar}}{8r^2}$ [(14B–2A)r– $(3B^2 - A^2 - 2AB + 16VC^2)r^2 + + 4(e^{\frac{(A+B)r}{2}}$ –1)] ………………(2.21)

$H_1^2\, e^{-2M} = \frac{1}{2V^2}\{e^{\frac{(A+B)r}{2}} - 1\} + \frac{1}{16}(3B^2 - A^2 - 2AB - 16VC^2) - \frac{1}{4r}(A+B)$
…………………(2.22)

$4\pi\sigma = \left\{\frac{d}{dt}F^{41} + \left(\frac{2}{r} + A\right)F^{41}\right\} e^{\frac{Br}{2}}$ ….(2.23)

$4\pi\,\eta = 2F\,\frac{e^{-Ar}}{Vr}$ …………………….…(2.24)

We see that p , σ, η are all become infinite at the center r = 0. So there is singularity in these quantities at center. For λ = ζ = τ = 1,

$2L = A\,r^2$ ……………………(2.25)
$2M = B\,r^2$
$S = Cr^2$

Now with the help of the (2.24), (2.8) to (2.12) mass density, pressure, charged density and scalar charge density can be calculated as in case λ = ζ = τ = 0. At r = 0 , there values are

$8\pi\rho_0 = \frac{1}{4}(17B - 7A)$ …...………………..…(2.26)

$16\pi p_0 = \frac{3}{2}(A+B)$……………………..…….(2.27)

$4\,\pi\sigma_0 = (3B^2 - A^2 - 2AB + 16VC^2)^{\frac{1}{2}}$ ..…(2.28)

$4\,\pi\,\eta_0 = 6\,V^{-1}C$…………………………….(2.29)

Next we assume that the constant L and M are positive. Hence ρ_0 and p_0 are both positive if $B > \frac{7}{17}$ A. Also σ_0 is real if

$3B^2 - A^2 - 2AB + 16VC^2 > 0$ ………….(2.30)

The boundary r = a at which p = 0 will satisfy the equation.

$\frac{3}{2}(A+B)\,a^2 - e^{(A+B)\frac{a^2}{2}} + 1 = 0$………..(2.31)

The geometric mass of the sphere is

$$B = \int_0^a \int_{\theta=0}^{\pi} \int_{\o=0}^{2\pi} \rho \, dv \ldots\ldots\ldots\ldots(2.32)$$

where dv the proper elementary volume.

$$dv = r^2 \, e^{2L-3M} \sin\theta \, d\o \, dr \ldots\ldots\ldots(2.33)$$

Thus the geometric mass of the sphere using (2.15) (for $\lambda = \zeta = \tau = 1$) and (2.32) in (2.31) is given by

$B = \int_0^a \left[\frac{1}{4} (17B - 7A) + (\frac{3}{2} A^2 - \frac{9}{2} B^2 +$

3AB)r2e−Mr2dr)+ 14

$\int_0^a (2e^{-2M} H_1^2 + 6VS_1^2) \, e^{-M} r^2 \, dr = \psi(a) + \omega(a) +$

S(a)………………(2.34)

Thus the mass density, electric charge and scalar charge density contributes to the geometrical mass of the charged fluid spheres is evident from (2.34).

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